Misallocation in the Market for Inventors *

Fil Babalievsky

December 8, 2023

Abstract

How much misallocation is there in the market for inventors? This paper introduces a novel growth model where firms search for inventors in a frictional labor market, and where innovation depends on the quality of the match between the inventor and firm. The model features both knowledge spillovers and congestion in the labor market, hence search effort can be either too low or too high. I document that inventors patent 40% more after moving across firms, suggesting that they had been mismatched prior to the move. I find that shutting down search frictions for inventors increases growth from 2.0 to 2.6 percent, that firms invest too little in searching for inventors, and that conventional R&D subsidies do not reduce misallocation. Finally, I show that partial-equilibrium effects of R&D subsidies overstate their general equilibrium effects because when firms innovate and creatively destroy their rivals' product lines, they also break their matches with inventors.

JEL codes: E6, J6, O3

Keywords: Growth, Innovation, Misallocation, Search

*US Census Bureau. I thank Kyle Herkenhoff, Loukas Karabarbounis, Jeremy Lise, Lee Ohanian, and Michael Peters for their support and feedback. I thank Jacob Adenbaum, Cristián Aguilera Arellano, Mariacristina De Nardi, Niklas Engbom, Martín García-Vázquez, Eugenia Gonzalez-Aguado, William Jungerman, Hannes Malmberg, Thomas May, Marta Morazzoni, Jo Mullins, Jane Olmstead-Rumsey, Jordan Pandolfo, Luis Pérez, Marta Prato, Nicolò Russo, Kjetil Storesletten, and the participants of the De Nardi-Lise-Mullins and Herkenhoff-Karabarbounis-McGrattan workshops at the University of Minnesota for many helpful conversations. All errors are my own. The views expressed in this paper are my own and do not necessarily reflect the position of the Census Bureau.

Introduction

This paper studies the labor market for inventors. Which inventor works for which firm can affect both firm performance and economic growth, as innovative activity combines inventor skills with the intangible capital of their firm. Hence, it does not come as a surprise that many high-profile inventors make headlines when they moved across firms (among them Peter Rawlinson from Tesla to Lucid and Jeffrey Wilcox from Intel to Apple and back)[1]. In this paper I propose a novel growth model that features a frictional labor market for inventors and use it to show that the allocation of inventors across firms has first-order consequences for growth and innovation policy.

I make three contributions. The first is theoretical: I introduce a new model in which firms engage in costly search for inventors, and where the rate at which an inventor-firm pair innovates is determined by their "match quality". Inventors engage in on-the-job search and therefore climb the match quality ladder, improving their rate of innovation. A successful innovation allows the firm to expand into new markets, rendering the incumbent producer obsolete. Crucially, this leads to a novel implication of creative destruction: the match between the incumbent and their own inventor breaks.

My model builds on both the growth and search literatures, and it inherits some of the efficiency properties of both. As in most models of growth, firms benefit from knowledge spillovers, and therefore do not internalize the benefit to other firms of making better matches and innovating more quickly. Search frictions can, as usual, mean that firms spend either too little or too much to meet with workers. On the one hand, firms do not internalize the benefit of their search effort to inventors (through higher wages) or to other firms (through knowledge spillovers). On the other hand, firms do not internalize the harms they cause to other firms by stealing their inventors. Crucially, my model features a novel spillover that is missing in both the search and growth literatures by themselves: when firms innovate and expand into new product lines, they do not internalize the fact that they break matches between incumbent firms and their inventors. This is the "Blackberry effect": when Blackberry's smartphones were creatively destroyed by Ap-

[1] Details about Rawlinson and Wilcox's stories can be found here and here, respectively.

ple's iPhone, many of its engineers lost their jobs. [2]

My second contribution is to document a set of novel facts from disambiguated patent data from the USPTO to discipline the key parameters of my model: the efficiency of the meeting technology and the difference between the best and worst match qualities. The first parameter governs the *extent* of misallocation, the second governs its *cost*. To discipline the extent of misallocation, I first show that inventors patent at roughly 1.6 firms on average, which suggests a significant amount of turnover given that the average time between an inventor's first and last patent is less than six years.[3] To discipline the costs of misallocation, I use a matched-pairs design related to that in Prato (2022): I compare inventors who move across firms to a "twin" inventor who never moves but otherwise appears nearly identical. I find that the inventor who moves has about 40% more patents going forward. This is not driven by moves to firms that systematically patent more, and continues to hold when I weight patents by their citations and number of coauthors. The fact that inventors see such large gains from mobility suggests that misallocation is very costly.

My third contribution is quantitative: I show that my model has a novel set of implications for growth and innovation policy. First, I use the estimated model to quantify the impact of the misallocation of inventors on growth. I find that removing this misallocation altogether could raise growth from 2.0 percent to 2.6 percent, which shows that search frictions in the market for inventors are costly for growth. Intuitively, the model finds such large costs because inventor mobility is both very frequent and also associated with an increase in patenting. Next, I use the model to conduct a series of counterfactuals. First, I ask whether firms are investing too much or too little in search and find that they invest far too little. Second, I test whether conventional R&D subsidies such as the R&D tax credit can address these inefficiencies by increasing firms' incentives to search. Strikingly, I find that R&D subsidies have minimal effects on average match quality or the share of matched inventors. The reason is the aforementioned "Blackberry Effect": all innovation in the model is creative destruction[4], and that an inventor working for a creatively-

[2] This news article describes some of their stories.

[3] I only focus on the part of an inventor's career when they are actually patenting, which helps explain why I find more mobility than Akcigit and Goldschlag (2023).

[4] Because of how I calibrate the model, this assumption is not quantitatively important. I target the rate at which young firms exit to match the rate of creative destruction, and the size of

destroyed product line loses their job. Greater subsidies for research give firms stronger incentives to search for inventors and spur innovation, but at the aggregate level an increase in innovation also means an increase in the rate at which "match capital" is destroyed.[5] I find that these two forces cancel each other out almost perfectly in general equilibrium. Finally, I show that this model has implications for the effectiveness of R&D subsidies. The partial equilibrium increase of a firm's innovation in response to an R&D tax credit, as one might estimate in a quasi-experiment, is partly driven by improved match quality, but this mechanism vanishes general equilibrium. Hence, the general equilibrium effect of R&D subsidies on innovation is only half as large as the partial equilibrium effect. Modeling the process by which firms match to inventors is therefore important for translating partial-equilibrium estimates into general-equilibrium predictions about the impact of innovation policy.

Literature and Contribution

This paper builds on and contributes to several literatures. First, this paper introduces a model of endogenous growth and firm dynamics in the tradition of Klette and Kortum (2004) or Peters (2020). A standard feature of such models is that firms can hire inventors on a frictionless spot market. My contribution is to instead model a frictional market for inventors and show that such a market structure has markedly different and empirically relevant consequences for growth and innovation policy.

This paper also contributes to the literature on frictional search by applying it to inventors and disciplining it with patent data. Other papers bridging the search and Schumpeterian growth traditions are Engbom (2020) and Lentz and Mortensen (2012), but these papers model frictional search for production workers as opposed to inventors.

A growing line of research uses patent data to study inventors. van der Wouden and Rigby (2020) study inventor mobility from 1900 to 1975 and also finds that in-

each innovative step to match the aggregate growth rate. If I introduced own-innovation or new product lines but disciplined creative destruction similarly, it would imply a smaller step size per innovation but the same rate of creative destruction.

[5]This is in addition to the dampening effects of creative destruction studied in Peters (2020), where a faster rate of creative destruction lowers the net present value of flow profits from an innovation.

ventor mobility is associated with an increase in patenting. I perform a similar empirical exercise and use it to discipline a structural model and perform policy counterfactuals. Akcigit and Goldschlag (2023) also study inventor mobility and finds that, among the sample of moves, inventors who move to young firms patent more than inventors who move to incumbents. Prato (2022) and Arkolakis, Peters, and Lee (2020) show that the migration of inventors across nations is a major factor in driving growth, whereas my paper studies the smaller-scale but more frequent mobility of inventors across firms. Akcigit, Caicedo, Miguelez, Stantcheva, and Sterzi (2018) study the allocation of inventors across research teams in a model where ideas are sold on a competitive market. Prato (2022) and Akcigit et al. (2018) abstract from firm dynamics to focus on inventor teams, whereas I abstract from teams to focus on the allocation of inventors across firms. Celik (2023) finds evidence that individuals may be misallocated between invention and production.

Finally, this paper contributes to the literature on innovation policy, in the vein of Atkeson and Burstein (2019). Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018) study R&D policy in a model that has heterogeneous and dynamic firm types but frictionless markets for homogeneous skilled labor. Rao (2016) studies the impact of R&D subsidies in partial equilibrium with a quasi-experimental method. Lashkari (2023) shows that firm entry and exit decisions also generate misallocation of innovative resources across firms and that there is a role for policy to correct these inefficiencies.

This paper is structured as follows. Section 1 introduces the model. Section 2 explains the data and facts I use to discipline the model. In Section 3, I merge the two and explain the calibration. In Section 4, I use the model to perform a series of counterfactuals. Section 5 concludes.

1 Model

Time is continuous. There is a stand-in household with a measure M of workers, who inelastically supply their labor to profit-maximizing firms. An exogenous share ν of these individuals work in the frictional market for inventors,[6] and

[6] I denote this share ν of individuals as "inventors" regardless of whether they are currently matched or not. The assumption of an exogenous measure of inventors is a reasonable simplification in light of the findings in Bell, Chetty, Jaravel, Petkova, and Van Reenen (2019) that the

the remaining share $1 - \nu$ can freely move across four frictionless sectors: final goods production, new firm creation, search for inventors, and creating the research good.[7] Inventors become non-inventors at a rate δ_i, and non-inventors become inventors at a rate $\delta_r = (\nu\delta_i)/(1-\nu)$.[8]

There is perfect risk-sharing in the household. The household derives logarithmic utility from the final good Y and discounts the future at rate ρ. Consumption of the final good is the only source of utility in this model and there are no aggregate shocks. Hence, we can write the household's discounted utility U as:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \log(Y_t) dt.$$

1.1 Production

The production side of the model is deliberately standard. As in Klette and Kortum (2004), the final good is a CES aggregate of an exogenous measure 1 of varieties. Each variety i may be produced by multiple firms f, and each firm may produce more than one good. Different varieties $i' \neq i$ are imperfect substitutes, with elasticity of substitution σ, but the same variety produced by different firms $f' \neq f$ are perfect substitutes. I find it convenient to denote $[fi]$ as the "division" within firm f that produces good i. Production of the final consumption good Y_t is as follows, where $y_{[fi]t}$ is the production of each division:

$$Y_t = \left(\int_0^1 \left(\sum_f y_{[fi]t} \right)^{\frac{\sigma}{\sigma-1}} di \right)^{\frac{\sigma-1}{\sigma}}.$$

Division $[fi]$ has access to a linear labor-only technology with productivity $z_{[fi]t}$:

$$y_{[fi]t} = z_{[fi]t} l_{[fi]t}.$$

I follow Garcia-Macia, Hsieh, and Klenow (2019) and assume that each division

children of inventors are far more likely to be inventors themselves. See Celik (2023) for a model that endogenously generates a fixed share of inventors.

[7] This is a stand-in for all inputs into innovation aside from inventors, from lab equipment to research assistants.

[8] This is consistent with a steady-state share ν of inventors and helps to rationalize the fact that most inventors only patent for a few years.

must pay an infinitesimal operating cost of ϵ to produce. This ensures that only the division with the highest z in each product line i will produce, and that it will charge a homogeneous markup $\mu = \sigma/(\sigma-1)$.[9]

I denote $z_{it} = \max_f z_{[fi]t}$ as the efficiency of the leading producer of product line i, $F_t(z_{it})$ as the CDF of product lines with efficiency z at time t, $Z_t = \left(\int_0^1 z_{it}^{\sigma-1} dF_t(z_{it})\right)^{\frac{1}{\sigma-1}}$ as the usual measure of "average productivity", and the "relative efficiency" of a division as $\hat{z} \equiv (z/Z)^{\sigma-1}$.

I denote the total mass of workers involved in production as l_t. Final goods output Y is $Z_t l_t$, wages for production workers w are $Y_t/(\mu l_t)$, and product-level gross profits π can be written as:

$$\pi_{[fi]t} = (\mu - 1) \underbrace{\left(\frac{z_{[fi]t}}{Z_t}\right)^{\sigma-1}}_{\text{Relative Productivity } \hat{z}} w_t l_t.$$

In what follows, I drop time subscripts where there is no possibility of confusion.

1.2 Innovation

Incumbent firms can only innovate if they match a division to an inventor. For tractability, I assume that each division within a firm can have at most one inventor.

Following Klette and Kortum (2004), I assume that all innovation in this model is creative destruction: when the inventor and division innovate, they draw a random product i', improve on its relative productivity $\hat{z}_{[fi']t}$ by λ, and spin off a new unmatched division within the same firm to produce the improved product line. I denote $N(\hat{z})$ as the value of an unmatched division with relative productivity $\hat{z}$, and $dF(\hat{z})$ as the PDF of product lines across relative productivity space. CD, the expected value of creatively destroying a product line, is:

$$CD = \int N(\hat{z}' + \lambda) dF(\hat{z}').$$

[9] If two "divisions" belonging to different firms f and f' can produce the same good i, and both decide to pay the operating cost, they compete in Bertrand fashion. If $z_{[f'i]t} < z_{[fi]t}$, then division $[f'i]$ will make no profits from production. Because of the operating cost, the less-productive division will not enter, and the more-productive division will charge a fixed markup.

The rate of innovation is an increasing function of the resources r (in units of labor) which the division provides to the inventor. Research effort r is a stand-in for all kinds of R&D expenditures (lab technicians, equipment, etc). The rate of innovation is also increasing in the time-invariant division-inventor match quality $q_{[fi]j}$, which acts as TFP for the innovation process. I assume that q is bounded above by q_h and below by q_l.[10] This notion of match quality is new to the literature.

The rate of incumbent innovation ϕ_n depends on match quality q and the quantity of the research good r that the firm provides:

$$\phi_n(q, r) = \underbrace{q}_{\text{Match Quality}} \cdot \underbrace{r^{\gamma}}_{\text{R\&D}}.$$

I denote $V(\hat{z}, q)$ as the value of a matched division with relative productivity $\hat{z}$ and match quality q. Note that $\hat{z}$ does not show up in the value of innovation CD, hence the benefit of being matched to an inventor is constant across all values of $\hat{z}$.

I assume that inventors and research spending cannot be transferred across divisions within the same firm. Combined with the fact that each variety is infinitesimal, this means that each division within a firm can make decisions without reference to any other division and that firm values are additively separable in division values. Without this assumption, the firm's state space would include its full list of product lines and inventors. For firms such as Google or Amazon that make very diverse products, this assumption is reasonable. Besides, a high degree of mobility within a firm is incompatible with Gibrat's law, as larger firms would have better matches and faster rates of innovation.

1.3 Compensating Inventors and Investing in Research

To model the bargaining between the division and the inventor, I assume the inventor and division can successfully bring the idea to market with probability 1, or the inventor can exercise their outside option and successfully bring the idea to market with probability $\chi < 1$. This captures the possibility that the inventor may try to steal the idea. If the division offers the inventor a share $\chi + \epsilon$ of the value of the patent, the inventor will always give the patent to the firm and it will always

[10] This captures the distance between the inventor's knowledge and the technical details of the product line they are building on, similar to the "circle of knowledge" in Jones (2009).

be brought to market. Hence, the firm always pays the inventor a share χ of the value of their innovation. I refer to χ as the "bargaining weight" of the inventor. [11]

Divisions therefore anticipate that inventors will keep some share of the value of innovation, which induces a hold-up problem for the division. The division therefore chooses r to maximize the following:

$$\max_r (1-\chi) q r^{\gamma} C D - w r.$$

1.4 Matching to Inventors

I now explain how inventors are matched to divisions. To look for inventors, divisions hire headhunters h from the mass of non-inventors, paying them a wage w. Effective search effort is $m(h) = h^{\eta}$, and if $\eta < 1$ this implies diminishing returns to search at the level of the division.[12] Divisions can search regardless of whether they are already matched or not, so they have the option to fire their current inventor and replace them if they find a better match. Search is random, and matched inventors are just as likely to be contacted as unmatched. Unmatched inventors earn no flow payments.

The meeting technology is a Cobb-Douglas function of aggregate firm search effort S and the total mass of matched and unmatched inventors νM, with a search efficiency parameter α (effectively the TFP of the matching technology) and an elasticity with respect to search effort θ.

$$H = \alpha \left(S\right)^{\theta} \left(\nu M\right)^{1-\theta}.$$

I denote $dF^N(\hat{z})$ as the mass of unmatched divisions with relative productivity

[11] This wage-setting protocol is similar a typical Nash bargaining setup with bargaining parameter χ, with the key difference being that it breaks the link between compensation and the expected duration of the match. This link is the major source of difficulty in using Nash with on-the-job search (see Shimer (2006) for details). My approach still avoids the need to carry around compensation as extra state, and as I will show later, it also helps avoid the need to solve for the inventors' value functions altogether. This assumption can be relaxed, but only at great computational cost.

[12] Assuming diminishing returns is analogous to introducing convex vacancy posting costs in a standard search model. As Kaas and Kircher (2015) show, this is empirically relevant, and it simplifies the model computationally. Also, without such an assumption, matched divisions would never search. With a linear search technology, unmatched divisions would be exactly indifferent between searching and not searching, and because matched divisions can set r to an arbitrarily small value they are strictly better off than if they were unmatched.

$\hat{z}$; and I denote $dF^V(\hat{z}, q)$ as the mass of matched divisions with relative productivity $\hat{z}$ and match quality q. Because innovation is unrelated to $\hat{z}$, so are the returns to search. I denote m^* and $m^*(q)$ as the optimal search effort of unmatched divisions and matched divisions with match quality q, respectively. Hence, aggregate search effort S is:

$$S = \int_{\hat{z}} m^* dF^N(\hat{z}) + \int_{\hat{z},q} m^*(q) dF^V(\hat{z}, q).$$

Upon meeting, the division and inventor draw a permanent idiosyncratic match quality q. They then decide whether to break any current match that they have and match with each other. If both agree to the match — that is, if they are both in each others' acceptance sets — the match retains its quality q throughout its duration.

The assumptions that I have imposed on inventor compensation and matching (no earnings when not matched, a fixed bargaining weight χ, and equal meeting rates for employed and unemployed) make the model far more tractable. Crucially, they allow me to solve for the acceptance sets and the firms' search policies without solving for the inventors' value functions. To show this, I begin with the following helpful proposition:

PROPOSITION 1. Assume that inventors and divisions only observe their own current and potential match quality q, and do not observe whether the other division or inventor is already matched. If an inventor and division meet and draw potential match quality q then:

1. If both are unmatched, the meeting converts to a match.

2. If only one party is matched, the meeting converts to a match if and only if the new match quality q is higher than the quality of the party's current match.

3. If both are matched, the meeting converts to a match if and only if the new match quality q is greater than the quality of both party's current matches.

First, recall that inventors earn no unemployment benefits if unmatched, and that the search technology is equally good for matched or unmatched inventors and divisions. A matched division has the option to set $r = 0$ and search as intensely as if it was unmatched, and it has the option to accept and reject the same potential matches as if it was unmatched. Hence, being matched is at least weakly better than being unmatched for a division. We can go further and show that this dominance must be strict: if $0 < \gamma < 1$, then as $r \to 0$ the division's profits from innovation $(1-\chi)qr^{\gamma}CD - wr$ become positive. Hence, a matched division can do strictly better than an unmatched one by keeping the same search-and-match policy but engaging in a small amount of research.

Because a matched division will always engage in a positive amount of research, it will always pay out a positive amount in compensation to the inventor in expectation. A matched inventor can do strictly better than an unmatched one by accepting and rejecting the same future offers but taking the division's compensation in the meantime. Therefore, both divisions and inventors strictly prefer being matched to being unmatched.

Because both the argmax and maximum of $(1-\chi)qr^{\gamma}CD - wr$ are increasing in q, divisions and inventors both strictly prefer high q to low q. Hence, they will leave a lower-q match if and only if a higher-q match is available.

Recall that the *only* decision inventors need to make is whether to accept a match or not. Hence, characterizing the acceptance sets of both the firms and inventors using Proposition 1 allows me to solve and estimate the model without solving for inventor value functions. With this proposition in hand I can now characterize the solution to the firm's problem of choosing its search effort.

I next introduce some helpful notation. First, it will be useful to write out the "contact rates", or the number of meetings per unit of search effort and the number of meetings per inventor, which I respectively denote C_s and C_i:

$$C_s = \frac{H}{S}, \quad C_i = \frac{H}{\nu M}.$$

I next introduce the notation Ω for the expected value of meeting a new inventor. Recall that each division can only match to one inventor, so the improper integral $\int_{\hat{z}',q'} dF^V(\hat{z}', q')/(\nu M)$ is the share of inventors who are matched. Ω is as follows for an unmatched division:

$$\Omega = \underbrace{\left(1 - \int_{\hat{z}',q'} \frac{dF^V(\hat{z}',q')}{\nu M}\right)\left(\int_q \left(V(\hat{z},q) - N(\hat{z})\right) dF^q(q)\right)}_{\text{Meeting Unmatched Inventor}}$$
$$+ \underbrace{\int_{\hat{z}',q'} \int_{q>q'} \left(V(\hat{z},q) - N(\hat{z})\right) dF^q(q) \frac{dF^V(\hat{z}',q')}{\nu M}}_{\text{Meeting Matched Inventor}}.$$

For a matched division with states $\hat{z}, q$, this instead becomes:

$$\Omega(q) = \underbrace{\left(1 - \int_{\hat{z}',q'} \frac{dF^V(\hat{z}',q')}{\nu M}\right)\left(\int_{q>\tilde{q}} \left(V(\hat{z},q) - V(\hat{z},\tilde{q})\right) dF^q(q)\right)}_{\text{Meeting Unmatched Inventor}}$$
$$+ \underbrace{\int_{\hat{z}',q'} \int_{q>\max[q',\tilde{q}]} \left(V(\hat{z},q) - V(\hat{z},\tilde{q})\right) dF^q(q) \frac{dF^V(\hat{z}',q')}{\nu M}}_{\text{Meeting Matched Inventor}}.$$

For unmatched and matched divisions, respectively, the problems of choosing effort are:

$$\max_h m(h)\Omega C_s - wh.$$

$$\max_h m(h)\Omega(q) C_s - wh.$$

Matches also break at an exogenous rate δ_m.

1.5 New Firm Creation

There is free entry into new firm creation: any of the $M(1-\nu)$ non-inventors can at any time decide to try to start up a new firm, thereby converting their endowment of time into a flow rate of new firms. The rate ϕ_s at which a worker can start up firms depends on the quality of the entry technology ι and the degree of congestion $\psi > 0$ in entry: if a measure E of new firms are created, the productivity of the entry technology is given by:

$$\phi_s = \iota / E^{\psi}.$$

Upon successfully entering, the entrepreneur[13] draws a random product line i and improves on the relative productivity $\hat{z}_{[fi]t}$ of the leading producer of i to $\hat{z}_{[fi]t} + \lambda$, as inventors do. The entrepreneur creates a new firm f' with a single unmatched division to produce i. I assume that the "creatively destroyed" division $[fi]$ exits and lays off their inventor.[14] The representative household's valuation of the new startup is the net present value of its future profits.

Free entry implies that:

$$\phi_s CD = w.$$

The mass of workers e in entry must also be consistent with the level of entry E:

$$\phi_s e = E.$$

The aggregate rate of innovation per product line is equal to the rate of creative destruction, summed across entrant and incumbent innovation. I denote it as Δ:

$$\Delta = \phi_s e + \int_{\hat{z},q} \phi(q) dF^V(\hat{z}, q).$$

1.6 Hamilton-Jacobi-Bellman Equations

In what follows I consider a transformed version of the economy on a balanced growth path: wages w and average productivity Z grow at a constant rate g, the distribution of divisions across relative productivities $\hat{z}$ and match qualities q is constant, and policies are constant.

The value of an unmatched division is given by these five parts: flow profit from its product, the possibility of creative destruction, the capital gains from drifting down in "relative productivity", the gains from meeting inventors Ω, and the costs of generating those meetings.

[13]The entrepreneur is the model analogue to Steve Jobs; the inventor is the analogue to Steve Wozniak.

[14]This simplification greatly increases the tractability of the model and can be microfounded if I assume that the frontier division, armed with the knowledge of how to produce i, can copy any innovation made by the laggard, reducing their expected profits from innovation to zero.

$$\rho N(\hat{z}) = \max_{h} \underbrace{\pi(\hat{z})}_{\text{Flow Profit}} - \underbrace{\Delta N(\hat{z})}_{\text{Creative Destruction}} \underbrace{-g(\sigma-1)\frac{\partial N(\hat{z})}{\partial \hat{z}}}_{\text{Drift Down}} + \underbrace{m(h)\Omega C_s - wh}_{\text{Search}}.$$

The value of a matched division has six parts: flow profits, the risk that the match breaks, the possibility of creative destruction, drifting down, innovation, and the risk that their inventor will meet another division. The risk of an inventor meeting a division with match quality q is increasing in $m^*(q)$, the equilibrium search effort of such a division.

$$\rho V(\hat{z},q) = \max_{r,h} \underbrace{\pi(\hat{z})}_{\text{Flow Profit}} - \underbrace{(\delta_m+\delta_i)(N(\hat{z})-V(\hat{z},q))}_{\text{Match Break}} - \underbrace{\Delta V(\hat{z},q)}_{\text{Creative Destruction}} \underbrace{-g(\sigma-1)\frac{\partial V(\hat{z},q)}{\partial \hat{z}}}_{\text{Drift Down}}$$

$$+ \underbrace{m(h)\Omega(q)C_s - wh}_{\text{Search}} + \underbrace{(1-\chi)qr^{\gamma}CD - wr}_{\text{Innovation}}$$

$$+ \underbrace{\int_{q'>q} C_i \left(N(\hat{z}) - V(\hat{z},q)\right) \frac{\int_{\hat{z}} m^* dF^N(\hat{z}) + \int_{\hat{z}}\int_{q_l}^{q'} m^*(\tilde{q}) dF^V(\hat{z},\tilde{q})}{S} dF^q(q')}_{\text{Inventor Finds New Division}}.$$

The last term deserves elaboration: inventors are contacted at rate C_i, and if they receive a match quality $q' > q$ they will be willing to leave. However, matched divisions will only be willing to take the worker on if their own match quality is less than q'. The set of firms that are willing to accept the worker, conditional on a sufficiently good match, comprise a share $\left(\int_{\hat{z}} m^* dF^N(\hat{z}) + \int_{\hat{z}}\int_{q_l}^{q'} m^*(\tilde{q}) dF^V(\hat{z},\tilde{q})\right)/S$ of overall search effort and thus of the set of firms that contact the worker.

1.7 Equilibrium

A balanced growth path consists of values V, N, policies $e, h, r,$, wages w, distributions F, F^V, F^N, and a growth rate g such that divisions maximize profits, workers maximize their utilities, and policies are consistent with the distributions and growth rate being constant over time.

2 Empirical Analysis

In order to estimate my model, I target a series of novel moments about the mobility of inventors across firms and patenting conditional on such moves. My main source of data is PatentsView, the US Patent Office's digitized repository of all US patent applications since 1976. A major value-added of PatentsView relative to the raw patent data is their effort to disambiguate the names of inventors and the individuals or organizations to whom the patents are assigned: the raw data only includes alphanumeric names rather than a consistent identifier.[15]

The key patent-level variables that I use are the inventor, the assignee (the firms and individuals who were assigned ownership rights to the patent), and the filing date of the patent (I use filing rather than grant dates as the former are presumably closer to when the patent was created.)

I restrict my analysis to only include US-based inventors, and I only focus on patents with at least one corporate assignee. I therefore do not need to be concerned with whether non-US patents or "backyard" patents are comparable to US corporate patents. Excluding "backyard" innovation also means that my dataset is more comparable to my model, which does not have such innovations. My sample ultimately includes 3.3 million unique patents, 1.4 million unique inventors, and 200,000 unique assignees.

2.1 Key Moments

In this section I introduce some of the moments that I use to discipline my model. In particular, these moments help me discipline the parameters that govern the dynamics of the market for inventors: the efficiency of matching α, the share of the population that can be an inventor ν, the rate of matches breaking δ_m, the rate at which inventors lose their creativity and become non-inventors δ_i, and the gap between the best and worst matches q_h/q_l.

Inventors in my sample have, on average, 5.66 patents across 1.6 assignees, and their first and last patents are separated by an average of 5.7 years.[16] This is consistent with a move once every 3.5 inventor-years or so, indicating about as

[15]Details of their disambiguation methods can be found here.

[16]If an inventor patents once in 2014 and once in 2019, I record their "career length" as 5 years. If a patent has multiple corporate assignees, I only count one.

much mobility among inventors as among production workers.[17]

As I do not have access to the actual dates of employment for each inventor, I cannot directly measure job-to-job or employment-unemployment transitions. I must instead use the dates when their patents were filed. In some cases I find that inventors patent for firm A, then firm B, then A again in rapid succession. This suggests that an inventor's final patents at their old firm may be filed after their first patents at their new firm. Therefore, I code an employee as having moved in year t if the plurality of their patents were assigned to a different firm than the last time when they had non-zero patents.[18]

I compute two moments to help discipline the dynamics of the inventor labor market. First, as a proxy for transitions into unemployment, I calculate the average number of years between an inventor's first and last patent with the same assignee.[19] I find that this average time gap is around 1.8 years. Second, as a proxy for job-to-job flows, I calculate the share of patents granted to inventors whose assignee (employer) at year t is different from their employer in the last year when they had a nonzero number of patents. I find that this share is about 20 percent. Analogously, 50 percent of patents in year t are awarded to inventors who had not patented in $t-1$, including inventors who had never patented at all. I summarize these moments in Table 1.

2.2 Patenting and Mobility: A Matched Pairs Design

I follow Prato (2022) and discipline the effects of mobility on innovation using a matched-pairs design, pairing inventors that move with similar inventors who do not move. My units of observation are inventor-years where the inventor files for a patent. My treatment observations are years in which an inventor moved and had a patent, and my placebo observations are years in which inventors had a patent but did not move.[20] I calculate "moves" in the same way as in the last section: I

[17] US Bureau of Labor Statistics (2022) finds that Baby Boomers had 2.1 jobs from ages 45 to 54, and had many more job changes at younger ages.

[18] If an employee has one patent at Microsoft in 2012, then one patent at Apple in 2014, I code them as having moved in 2014. If an employee has one patent for Apple in 2012, then one for Apple and two for Google in 2014, I still code them as having moved in 2014 even if their patent at Apple was filed later in 2014 than their two patents at Google.

[19] I "zero-index" this count in both the model and the data. So an inventor who only patents for a firm in 2016 has a tenure of 0 years.

[20] I allow one inventor-year to act as a placebo for multiple treatments.

Table 1: Key Moments

Description	Value
Inventor career length (years)	5.7
Average patents per inventor	5.7
Average assignees per inventor	1.6
Average inventor-firm tenure (years)	1.8
Share of patents to new inventors	50
Share of patents to poached inventors	20

**Notes:* "Inventor career length" is the average number of years between an inventor's first and last patent. "Average patents per inventor" is the average number of unique granted patents that each inventor has, with coauthored patents treated the same as solo-authored patents. "Average assignees per inventor" is the number of unique corporate assignees in the patents granted to the inventors. "Average inventor-firm tenure" is the average number of years between the inventor's first and last patents at a given assignee. "Share of patents to new inventors" refers to the share of patents at time t awarded to inventors who did not patent in $t-1$. "Share of patents to poached inventors" refers to the share of patents at time t awarded to inventors who had patented for a different firm in the last year before t that I saw them in the data. See Section 2.1 for details.

code an inventor as having "moved" in year t if the plurality of the patents they filed in that year were assigned to a different firm than in the last year when they had nonzero patents.

As in Prato (2022) I match treatment years to placebo years based on the cumulative number of patents that both inventors had at $t-1$, the inventors' first year with a patent, the current year, and the fact that both had non-zero patents in year t. The last condition is important as it means that inventors patent in both the treatment and placebo years—that is, I am comparing treatment years with at least 1 patent to placebo years with at least 1 patent. I am not comparing individuals who move (and who, by definition, continue to patent) with individuals who stay but may stop patenting. Aside from these matching conditions, my large sample size gives me the opportunity to use stricter matching conditions than Prato (2022). I therefore choose my placebos from the list of inventors that never change firms, and I also match treatment to placebo based on their most recent employer before t and on the exact number of patents that both filed in the year $t-1$. Naturally, I also choose both treatment and placebo from the set of inventors who patent in more than one year. Ultimately, I am left with a sample of over 75,000 inventor-years with a move matched to a placebo.

Like Prato (2022) and unlike Akcigit and Goldschlag (2023), my treatment and placebo consist of inventors who continue to patent after a move. This implies that both individuals are still working as inventors, and is a more appropriate comparison with which to discipline my model: estimates of the gap between a good and bad match require comparisons between inventors whose careers are still continuing, not individuals whose innovative streak may have already ended.

Because my placebos are chosen from the set of inventors who stay at the same firm and keep patenting after $t-1$, the original firm must continue to patent after the treatment inventor leaves. This should exclude many instances when a "move" across firms was in fact a merger or acquisition.

All of my specifications report differences between the treatment T and placebo P, and all parameters are year indicators. I do not include $t-1$ in these specifications, as I match the treatment and placebo on their exact number of patents in that period. My regressions take the form:

$$Y_j^T - Y_j^P = \sum_{\tau \in [-5,-2] \cup [0,5]}^{5} \beta_\tau \mathbb{1}(j = \tau) + \epsilon. \tag{1}$$

I use four different outcomes for Y. In my first and primary specification, Y_j^i is the number of patents filed by i in relative year j. In my second specification, Y_j^i is instead the number of patents multiplied by their number of citations, divided by their number of authors. I refer to this as their "weighted" patent count. Multiplying by citations addresses the fact that not all patents are equally important, and dividing by the number of authors addresses the fact that I assign all patents to a single inventor and abstract from the role of teams. In my third and fourth specifications, I ask whether my results are driven by inventors moving to systematically more-productive firms, rather than movement along a match quality "ladder" as in my model. To do so, I take the same treatment-placebo pairs and replace their own patent counts (both raw and weighted) with the average number of patents per inventor at the firms that they worked at.[21] I show the results of this analysis in Table 2: mobility is associated with a large increase in the treated inventor's own patents but is not associated with an increase in the average patents of their firm. That is, inventors seem to become more productive when they move, but do not

[21]If an inventor patents at Apple in 2014 and Microsoft in 2016, I replace their patents with the average inventor-level patent count at Apple in 2014 and Microsoft in 2016, but keep them at 0 in 2015.

necessarily go to firms that systematically patent more. The same results hold for their weighted count of patents.

Table 2: Coefficients

	Raw Patents	Weighted	Firm Patents	Firm Weighted
	(1)	(2)	(3)	(4)
β_{-5}	0.004	0.408**	-0.080***	-0.300***
	(0.005)	(0.136)	(0.002)	(0.060)
β_{-4}	-0.003	0.338**	-0.067***	-0.161**
	(0.005)	(0.118)	(0.002)	(0.052)
β_{-3}	-0.020***	0.289**	-0.051***	-0.106**
	(0.004)	(0.102)	(0.002)	(0.040)
β_{-2}	-0.038***	0.136	-0.026***	-0.088***
	(0.003)	(0.087)	(0.001)	(0.023)
β_0	0.127***	3.361***	-0.157***	1.407***
	(0.007)	(0.189)	(0.003)	(0.092)
β_1	0.243***	2.656***	-0.271***	0.361***
	(0.008)	(0.128)	(0.004)	(0.076)
β_2	0.265***	2.572***	-0.254***	0.488***
	(0.008)	(0.144)	(0.004)	(0.094)
β_3	0.289***	2.306***	-0.241***	0.474***
	(0.008)	(0.140)	(0.004)	(0.068)
β_4	0.303***	2.405***	-0.241***	0.494***
	(0.008)	(0.129)	(0.004)	(0.073)
β_5	0.306***	2.198***	-0.242***	0.451***
	(0.008)	(0.108)	(0.004)	(0.063)
Estimator	OLS	OLS	OLS	OLS
N	604,647	604,647	604,647	604,647
R^2	0.014	0.004	0.053	0.001

* *Notes:* This table reports the results of Equation 1 for four different outcome variables Y: the inventors' own raw patent count, the inventors' count of "weighted" patents (i.e. multiplied by their number of cites and divided by their number of authors), and the average of the same two variables at the level of the firm. To calculate the firm-level variables, I replace the inventors' own patents or weighted with the average at the firm where they had most of their patents in that year. If they did not patent in a given year, I use the last firm I saw them patenting at. β_j is the coefficient on an indicator variable taking value 1 in year j, where j is defined relative to the treatment year. N refers to the number of observation-years, hence each treatment-placebo pair can appear up to 10 times (from $j = -5$ to $j = -2$, and then from $j = 0$ to $j = 5$). Because I match the treatment and placebo on the exact number of patents in $j = -1$, I drop that year from this regression. I cluster standard errors at the level of the treatment-placebo pair.

To help visualize and interpret the results more easily, I scale all coefficients and standard errors by the mean of Y_j^T for $j \in [-5, -1]$ and plot them in Figure 1. A coefficient of 0.4 means that the difference in Y between the treatment and placebo at time j was 40 percent of the treatment's mean value of Y in the pre-treatment period.

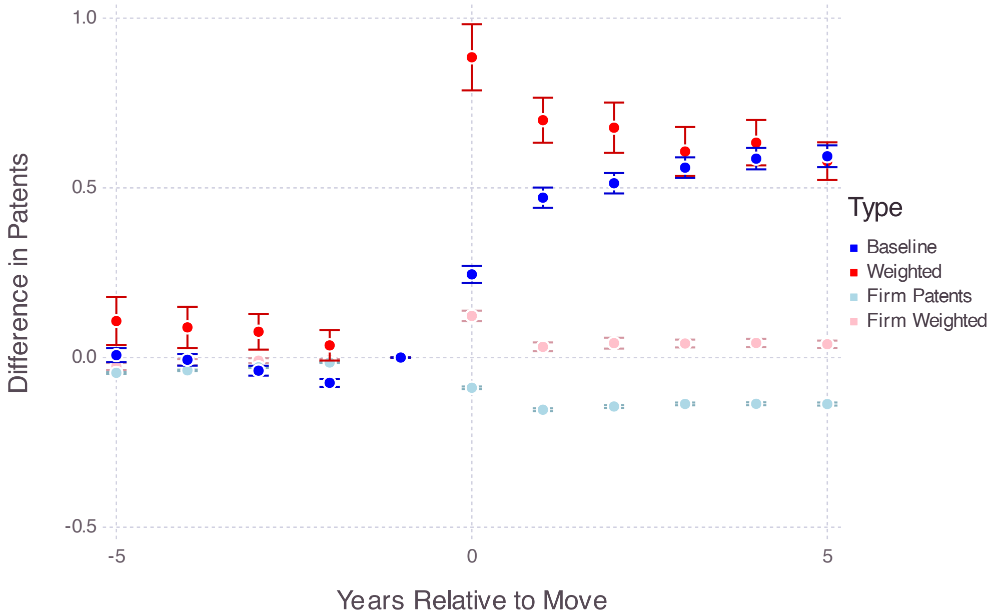


Figure 1: Difference in Outcomes, Treated vs Placebo

Notes: This figure illustrates the results of Equation 1 and Table 2. I again report the difference between the treatment and placebo inventors along four outcome variables Y: own patents, own patents-by-citations, and their firm-level counterparts. I scale the coefficients and standard errors by the mean value of Y for the treated inventors in the pre-treatment period. Hence, a value of 0.5 can be read as "the difference in Y between the treatment and placebo was 50% of the baseline value of Y for the treatment." See the notes for Table 2 for additional details.

Treated inventors patent more than placebos after they move, a pattern that is robust to scaling up the raw count of patents by the number of forward citations that they receive and then dividing by the number of coauthors. These patterns are not driven by the inventors moving to firms that systematically have more patents per inventor—if anything, the raw patent counts indicate that they move to firms that tend to have *fewer* patents per inventor. There is modest evidence that they do move to firms where inventors are slightly more productive per capita when we look at the weighted counts, but those coefficients are very small.

I also find that my results are driven by an increase in patenting among treated inventors, rather than a fall in patenting among the placebos. To show this, I calculate the ratio of patents in the pre-treatment period to patents in the post-treatment period for the placebo and treatment and demonstrate that both are well above 1:

$$\frac{\text{Mean Patents of Treatment, } t >= 0}{\text{Mean Patents of Treatment, } t < 0} \sim 1.73$$

$$\frac{\text{Mean Patents of Placebo, } t >= 0}{\text{Mean Patents of Placebo, } t < 0} \sim 1.18.$$

To summarize these results in a single number (which I replicate with simulated data and target as a moment), I calculate the average change in the percentage difference in patents between the treatment and placebo in Equation 2:

$$\frac{\text{Mean Patents of Treatment, } t >= 0}{\text{Mean Patents of Placebo, } t >= 0} - \frac{\text{Mean Patents of Treatment, } t < 0}{\text{Mean Patents of Placebo, } t < 0.} \tag{2}$$

I find that this number is around 0.44—that is, the average treatment inventor has the same number of patents as the average placebo inventor in the five years before the move, but has 44 percent more patents in the six years during and after the move.

3 Quantitative Exercise

I now explain how I map the model to data, with the goal of quantifying the losses caused by search frictions in the market for inventors and asking how or whether policy can recover those losses.

3.1 Calibration

I need to calibrate the elasticity of substitution across varieties σ, the rate of discounting ρ, the innovation curvature parameter γ, the inventor bargaining weight χ, the firm entry congestion curvature ψ, the elasticity of the meeting function θ, the parameters of the match quality distribution, the share of the population ν that can be an inventor, the exogenous match break rate δ_m, the efficiency of the search technology α, the step size λ, the efficiency ι of the firm entry technology, and the measure of workers M. I split these into externally and internally calibrated parameters and describe each in turn.

3.1.1 Externally-Calibrated Parameters

I set the elasticity of substitution σ and discounting ρ to standard values of 0.05 and 4 as in (for example) Peters (2020). I set $\gamma = 0.5$, a standard value on the response of innovation to research effort—see Section II.C of Acemoglu et al. (2018). My model includes both the research good r, which can be adjusted frictionlessly, and matches with inventors, which take time to adjust. Blundell, Griffith, and Windmeijer (2002), a key source of the consensus value reported in Acemoglu et al. (2018), uses a dataset with R&D expenditures over a relatively short timespan between 1972 and 1979. Hence, I interpret these estimates as capturing the short run response of r before matches adjust.[22] I set the elasticity of the meeting technology θ to a standard value of 0.5, as in Fujita and Moscarini (2017) or Gavazza, Mongey, and Violante (2018). I follow Peters and Zilibotti (2021) in imposing a small degree of congestion ($\psi = 0.1$) in the entry technology for firms. As in that paper, this is purely for computational convenience.[23] For the curvature of the match cost technology η, I follow Kaas and Kircher (2015) and set it to 0.5.

3.1.2 Internally-Calibrated Parameters

I match the remaining parameters by replicating key moments in both the data and the model. For a subset of moments, I calculate analogues using the steady state and do not simulate. For the remainder of these moments, I simulate a panel of 30,000 inventors over 40 years and calculate the same moments in the data as in the model. I estimate all parameters jointly but give a heuristic explanation of which moments map to which parameters in this section.

For the initial draw of match quality, I assume a uniform distribution with support $[q_l, q_h]$. I target q_h/q_l, the ratio of productivities of the most and least productive matches, so that the model can reproduce the 40 percent increase in patenting that I find with my matched-pairs design. I calculate the same moment reported in Equation 2 in the simulated data as in the real data. As in the data, I code an inventor as having moved in year t if the plurality of their patents in t were assigned to a different firm than in the last year when they had a patent.

[22] I study the long run effects of R&D subsidies in general and partial equilibrium, after firms have had time to adjust their search effort, in Section 4.3.

[23] This ensures that the amount of labor demanded for entry will never have a corner solution.

I scale the level of three parameters (the highest and lowest match qualities (q_h and q_l) and the entrant productivity ι) to ensure that GDP growth is 2 percent, and target the ratios of these three parameters with additional moments.

I use the level of the new-firm creation parameter ι (or, more precisely, the ratio between ι and q_l) to target the share of employment in firms less than one year old, which was roughly 1.4 percent in 2018 according to Hong and Werner (2020). Because an entrant firm consists of a single division and operates a single product line at first, I approximate this moment without simulation by multiplying the mass of entrants by the average employment in a newly-created division.

I set λ to target an exit rate of "new" (less than 1 year old) firms of 14 percent, a value I take from Peters and Walsh (2022). Growth in this model is equal to $\lambda\Delta/(\sigma-1)$, and the exit rate of entrant firms (which start with a single product) is approximately Δ. In order to reconcile 2 percent growth, an elasticity of substitution of 4, and a creative destruction rate of 14 percent, λ must be $3/7$.

To calibrate ν, the exogenous share of workers who are capable of being an inventor, I target the share of workers who patent in a given year. In the data, I divide the number of individuals with a patent in each of the years from 1976 to 2022 by the size labor force in that year and take the average of that value. On average, fourteen in every ten thousand workers have a patent each year.

I calibrate the match break rate δ_m to match the 1.8 years separating an inventor's first and last patent within the same firm.[24] To discipline the rate δ_i at which inventors lose their creativity and become non-inventors, I target the average length of time between an inventor's first and last patent.

I use α, the efficiency of the meeting technology, to target the 20 percent of patents in year t that were awarded to inventors who move to a new firm in t.[25] This is a rough proxy for the number of new matches in the economy and is therefore informative about the rate at which meetings happen.

To discipline the bargaining weight χ, I use evidence collected in Koh, Santaeulàlia-Llopis, and Zheng (2020) that labor comprises roughly 60 percent of the cost of investment in intellectual property. The closest analogue to these labor

[24] An inventor who only patents in a single year counts as 0 towards this mean. This is true for both the moment I calculate in the data and in the model.

[25] I use the same approach as in the matched-pairs design: I count an employee as having moved if the plurality of their patents in year t were awarded to a different firm than the last year when they had a patent, and I do this in both in the real and simulated data.

costs in the model are inventor salaries, as I interpret r as labor used to produce physical inputs for research. I therefore calculate the ratio of inventor pay to inventor pay plus the cost of r and target a value of 0.6.

I have normalized the measure of product lines to 1, but I cannot similarly normalize the measure of workers M. Growth in this economy roughly depends on the number of inventors per product line times the average number of innovative steps per inventor. M scales the first of these two numbers so that the model can rationalize the second while still producing 2 percent GDP growth.

3.1.3 Other Parameters

I add two additional "parameters" for the full quantitative implementation.

First, not every innovation is recorded as a patent, and my estimates will be biased if I map innovations directly onto patents. Arundel and Kabla (1998) find that 35 percent of innovations are recorded as patents in their sample of large European industrial firms, which I use as a benchmark. I therefore set $\kappa = 0.35$ as the share of ideas that are patentable, and only record a random 35 percent of innovative steps as patents in my simulated data.

Second, I add an R&D subsidy, which I model as a fixed percentage subsidy s_r for all expenditures on r workers and compensation to inventors. The problem of a matched division choosing r becomes:

$$\max_r (1 - \chi(1 - s_r)) q r^\gamma C D - w r (1 - s_r).$$

The government finances these subsidies in the model with taxes on the labor income of non-inventors. As non-inventors supply their labor inelastically, such taxes are non-distortionary, and I do not need to loop over taxes to balance the budget.

I treat s_r, the effective R&D subsidy, as a parameter to be estimated. Tyson and Linden (2012) report that in 2008, the R&D tax credit cost 11 billion dollars, and direct government support for corporate research cost another 26 billion. With US GDP at about 15 trillion that year — see US Bureau of Economic Analysis (2022) — I target a 0.0025 GDP share of corporate research subsidies.

3.2 Results

In Table 3 I present the results of the estimation. The model performs well on all targets, even though it has too few degrees of freedom to match all moments perfectly. Crucially, it does a good job of matching the increase in patenting that a “treated" inventor experiences upon a move. It attributes this to a spike in patenting for the mover rather than a decline in patenting for the stayer, which is also consistent with the data. The model finds that patents are worth about 7 times the wage of a non-inventor worker, which implies a value notably lower than the median in Kogan, Papanikolaou, Seru, and Stoffman (2017). However, Kogan et al. (2017) themselves admit that their patent valuations are likely skewed upwards due to functional form assumptions.

In the model, 15.5 percent of GDP is paid out as investments in innovation (compensation to r or to inventors). Atkeson and Burstein (2019) report a wide range of reasonable estimates for the right GDP share of such investments, from the 3 percent of value added in research and development on the low end to the estimated 13 percent of non-farm output in the broader measure of Corrado, Hulten, and Sichel (2009), with their preferred value of 6 percent of value added splitting the difference. The model's inferred value of investments in innovation is towards the upper range of plausible estimates, but is not unreasonable. As there is significant uncertainty about the true share of output that acts as an investment in innovation, I leave this as an untargeted moment. I report other properties of the equilibrium in Appendix C.

In Figures 2 and 3, I plot the results of running the regression specified in Equation 1 on real and model-generated data, respectively. The overall patterns are quite similar even though I target the moment detailed Equation 2 rather than the coefficients coming from 1. Note that, in both the model and the data, I condition on having at least 1 patent in the treatment year, but in the data authors often file multiple patents at the same time whereas this is not the case in the model. Hence, the difference in patents in the first year in the model is almost exactly zero by construction.

Table 3: Calibration

	Description	Value	Target	Data	Model
q_l	Lowest match quality	1.4	GDP growth	2%	2%
q_h	Highest match quality	1.73	Patenting change after move	44%	36%
			Average assignees per inventor	1.6	1.9
λ	Step size	0.43	Entrant exit	14%	14%
ν	Inventor share	0.002	Share of workers with a patent	0.14%	0.14%
δ_m	Exog. match break	1.1	Average inventor-firm "tenure" (years)	1.85	1.78
δ_i	Inventor exit	0.12	Inventor "career" lengths (years)	5.7	5.5
			Average patents per inventor	5.7	5.6
α	Search efficiency	9.4	Share of patents to "poached" inventors	0.2	0.22
			Share of patents to "new" inventors	0.5	0.52
χ	Inventor bargaining weight	0.54	Inventor pay/research costs	0.6	0.63
ι	Firm creation efficiency	0.03	Employment share of startups	1.4%	1.4%
M	Measure of workers	8.17	Average patents per year	1.0	1.0
s_r	Research subsidies	0.018	Subsidies/GDP	0.0025	0.0026
ρ	Discounting	0.05	Standard, i.e. Peters (2020)		
σ	Elasticity of substitution	4	Standard, i.e. Peters (2020)		
γ	Innovation cost curvature	0.5	Standard, i.e. Acemoglu et al. (2018)		
α	Meeting elasticity	0.5	Standard, i.e. Fujita and Moscarini (2017)		
η	Search cost curvature	0.5	Kaas and Kircher (2015)		
κ	Patentable share of ideas	0.35	Arundel and Kabla (1998)		
ψ	Entry congestion	0.1	Set externally—see Peters and Zilibotti (2021)		

**Notes:* This table reports the parameters for the calibrated model and compares model-generated moments with the data. All parameters affect all moments, but I list the parameters next to the moments which they most strongly affect. I take growth from the BLS, the rate at which new firms exit from Peters and Walsh (2022), the labor share of research costs from Koh et al. (2020), the entrant share of employment from Hong and Werner (2020), the share of GDP going to R&D subsidies from Tyson and Linden (2012) and US Bureau of Economic Analysis (2022). I calculate the rest of the internally-targeted moments from PatentsView.

3.2.1 The Allocation of Inventors

64 percent of potential inventors are matched in equilibrium. This surprisingly low number is an artefact of my choice not to let them take other jobs when not working as an inventor.[26] Recall that the minimum value of q is around 1.5 and the

[26]Without this assumption, it is possible that certain values of q would be unacceptably low for an unmatched inventor. This would make it challenging to separately identify α and the lowest value that q could take. As potential inventors account for a tiny share of all workers, this is

Figure 2: Change in Patenting: Data

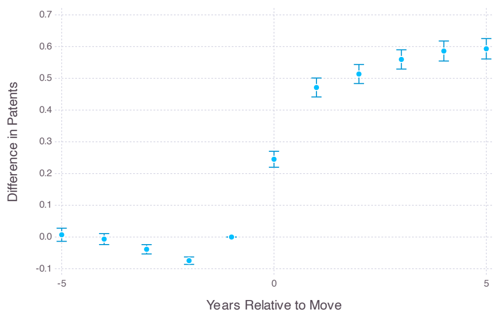
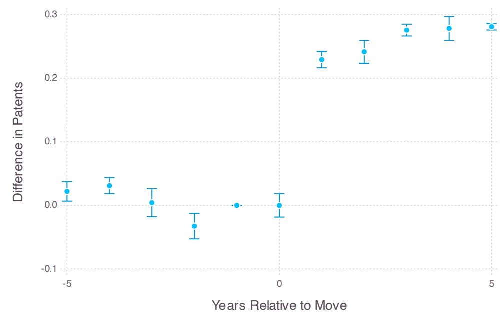


Figure 3: Change in Patenting: Model



Notes: Figures 2 and 3 report the results of running the regression specified in Equation 1 on real and model-generated data, respectively. I scale the coefficients and standard errors by the mean value of Y for the treated inventors in the pre-treatment period. Hence, a value of 0.5 can be read as "the difference in Y between the treatment and placebo was 50% of the baseline value of Y for the treatment." See the notes for Table 2 for additional details.

maximum is around 1.7. The average match quality conditional on being matched is 1.6, indicating that inventors are well-matched in general. In Figure 4, I display the distribution of matched inventors across the match quality distribution. When an inventor and firm meet, they draw their match quality from a uniform distribution. Because there is both on-the-job and replacement search, inventors and firms climb the "match quality ladder" and replace worse matches with better ones. In Figure 5, I show how the rate of innovation ϕ varies with match quality q. Because firms supply inventors with the research good r, and because the optimal value of r increases with match quality q, the rate of innovation ϕ grows faster than q. Still, the best possible match only innovates about 55 percent more than the worst.

4 Counterfactuals

In the next sections I study the costs of the misallocation of inventors, and ask whether policy can recover some of these costs.

4.1 The Cost of Search Frictions

In order to establish a benchmark or upper bound for the cost of the misallocation of inventors, I begin by increasing the efficiency of the search technology α until the average match quality q of a matched inventor gets close to the highest possible

quantitatively unimportant.

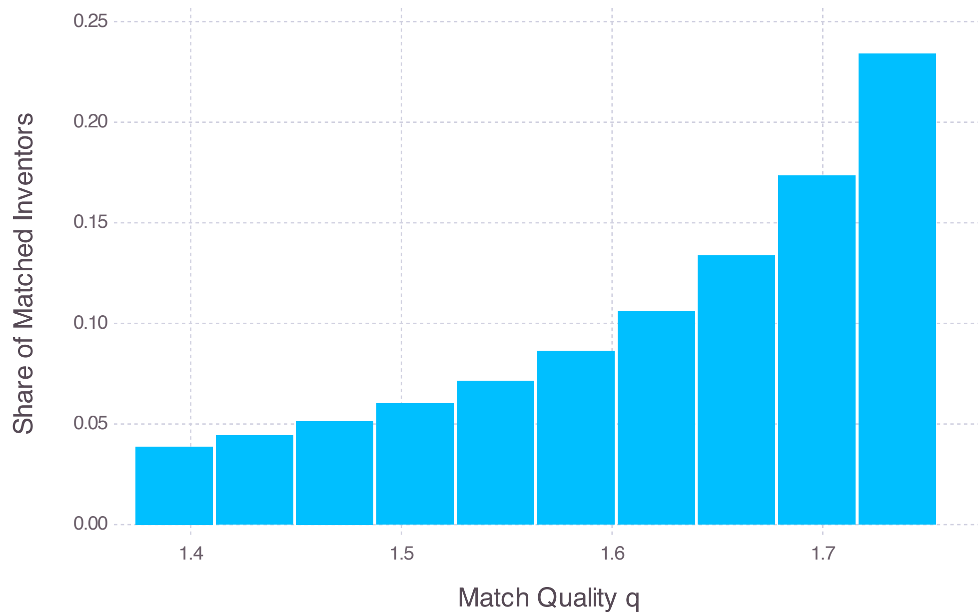


Figure 4: Distribution of Matched Inventors Across Match Qualities

Notes: This figure plots the share of matched inventors in each level of match quality. I discretize the model with 10 points on the match quality grid. Inventors and firms draw a match quality q when they meet, but because both inventors and firms look for better matches, they climb up the match quality ladder. Hence, almost a quarter of inventors have the best possible match quality, while less than 5 percent have the worst.

value and until almost all inventors are matched. This should not be interpreted as a policy counterfactual but as an upper bound for what policy can possibly do. I find that growth increases from 2.0 to 2.6 percent as search frictions disappear, an enormous gain with significant consequences for welfare. In Figure 6, I plot the growth rate, share of matched inventors, average match quality conditional on being matched, share of labor allocated to producing the research good r, and consumption equivalent welfare (all normalized to 1 in the baseline economy) as α increases and search frictions disappear. The dashed horizontal lines indicate the upper bounds of match quality and the matched share of inventors.

The share of labor allocated to producing the research good r does not scale up as quickly as the share of inventors who are matched. This is because a greater rate of innovation also means a greater rate of creative destruction, which lowers the returns to innovative effort and lowers the research intensity for a given inventor-

Figure 5: Match Quality and the Rate of Innovation

Notes: This figure plots the match quality q and rate of innovation ϕ for different match qualities. I rank match qualities from 1 to 10 on the horizontal axis, and plot their corresponding q and ϕ on the vertical axis after rescaling by the lowest value of both of those quantities. The rate of innovation scales up with both q and the quantity of the research good r that firms provide inventors, which itself is increasing in q. Hence, ϕ grows faster than q.

firm pair.

Even though reducing search frictions in the market for inventors would significantly enhance growth, it does not follow that the economy under-invests in inventor search h at the current value of α, or that there is a useful role for policy. The economy faces a simple dynamic tradeoff: On the one hand, a unit of labor that is allocated to the meeting technology increases the rate at which firms match with inventors, which will increase the average match quality q and therefore the growth rate of technology and consumption. Because of the quality-ladder structure of innovation, firms do not fully internalize the benefits of making a good match and innovating more quickly: some of the benefit goes to the rival firms that will creatively destroy their product lines. On the other hand, the labor used in the meeting technology cannot be allocated to producing the final good, so there is less consumption holding fixed the level of technology. Because there is conges-

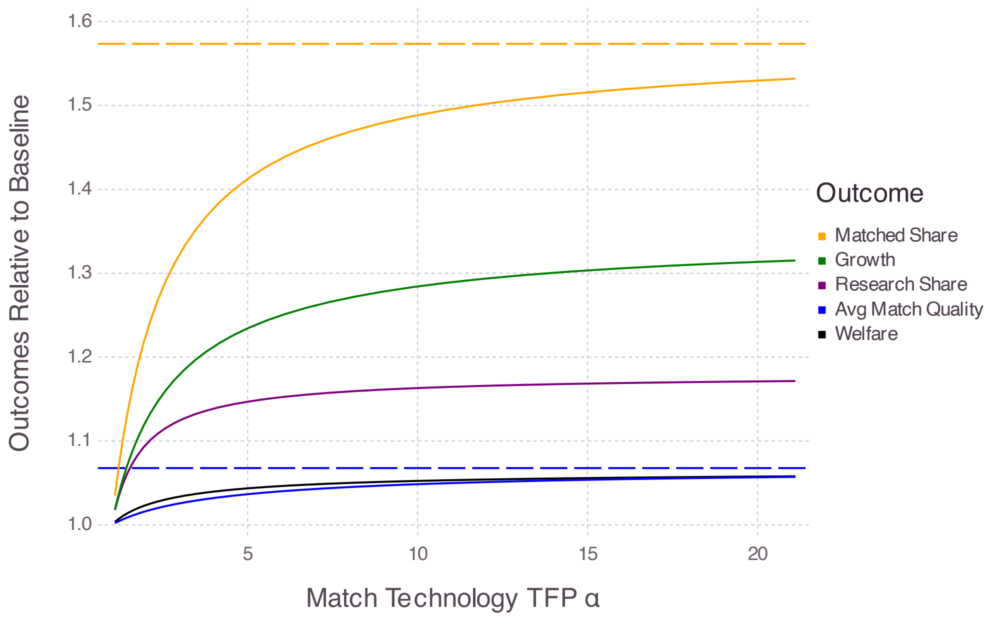


Figure 6: The Impact of Improving Match Efficiency

Notes: This figure plots the growth rate, share of matched inventors, average match quality conditional on being matched, share of labor allocated to producing the research good r, and consumption equivalent welfare as α, the efficiency of the meeting technology, increases. To plot variables on the same scale and to make interpretation easier, I normalize all quantities so that they are equal to 1.0 at the baseline calibration. Hence, a value of 1.01 corresponds to a 1 percent increase. The horizontal axis is the efficiency of the meeting technology, also normalized so that the baseline is equal to 1. The dashed horizontal lines indicate the upper bound of match quality and the matched share, and they have the same colors as the variables they map onto.

tion in the market for inventors, the resources expended in search may even be too high. In order to understand whether knowledge or congestion spillovers dominate, I introduce a “wedge" τ into the firm's decision problem for search effort and re-solve for the new distorted equilibrium. A positive wedge $\tau > 0$ induces firms to spend more than is privately optimal, and a negative wedge $\tau < 0$ induces them to spend less. These wedges leave the efficiency of search unchanged. The firms now maximize the following distorted problem:

$$\max_h m(h)\Omega H_s - wh(1-\tau).$$

In Figure 7, I plot the rate of growth, consumption-equivalent change in welfare, share of matched inventors, and match quality conditional on being matched as I increase and decrease the share of labor allocated to search for inventors. I find that increasing search effort would be welfare-improving, because the positive knowledge spillovers from more innovation through better-matched inventors dominate the negative spillovers from congestion.[27] I find that welfare is monotone-increasing for values of τ quite close to 1. This is in part because my calibrated search costs are extremely low and the share of the workforce allocated to search is tiny, so even increasing the number of workers devoted to search many times over only reallocates a small share of the overall workforce.

4.2 Can Policy Recover These Losses?

This model has two margins of investment: firms can invest in their "match capital" by increasing h, and they can invest in their "idea capital" by increasing r. I summarize the various subsidies to the latter form of investment as s_r, a stand-in for the R&D tax credits and other research subsidies. One might hope that R&D subsidies could bring the economy fairly close to the optimum by themselves: subsidizing r and inventors' salaries increases the returns to search effort, and can in principle indirectly raise the average match quality to the point where there would be minimal gains from any further improvements.

In practice, I find that the indirect effects of R&D on the allocative efficiency of inventors are largely canceled out by their indirect effect on creative destruction. This intuition points to a novel reason why the benefits of R&D subsidies may not scale up: the additional creative destruction coming from more innovation by rival firms destroys "match capital" and lowers the innovative potential of the economy. In contrast to standard Schumpeterian growth models, such capital is costly to build up and is lost to the economy when a division becomes obsolete.

In Figure 8, I plot the share of matched inventors and the average match quality of inventors conditional on being matched at all for R&D subsidies going all the way to 30 percent of GDP. Growth increases due to the large increase in r but match quality and the share of matched inventors stay basically flat. The share of matched

[27] As an aside, I find that actually implementing these wedges as subsidies, even for values of τ close to 1.0, costs the government far less than traditional R&D subsidies for a given increase in growth. This connects my paper to a long tradition in the search literature of considering whether there is "too much" or "too little" vacancy posting, dating back to the seminal contribution of Hosios (1990). I leave it to future work to figure out whether or how such subsidies could be implemented.

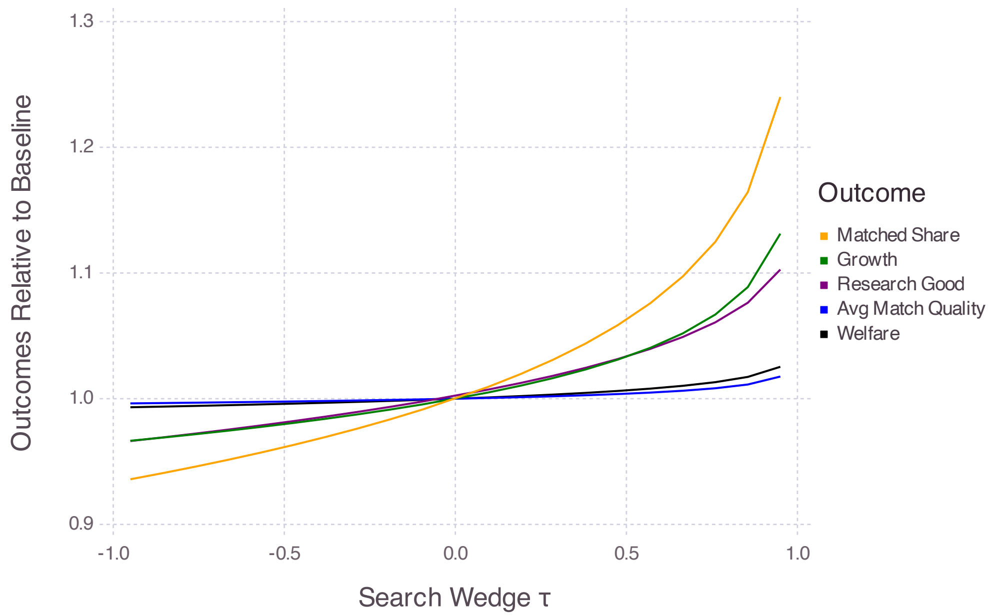


Figure 7: The Impact of Changing Search Effort

Notes: This figure plots the growth rate, share of matched inventors, average match quality conditional on being matched, share of labor allocated to producing the research good r, and consumption equivalent welfare under different values of the distortion τ. Firms decide on their search effort "as if" the price of search was $(1-\tau)w$ rather than w, meaning that positive values of τ cause firms to increase their search effort. The efficiency of search α is the same—this exercise asks whether, at the current level of search efficiency, firms invest too much or too little in search. To plot variables on the same scale and to make interpretation easier, I normalize all quantities so that they are equal to 1.0 at the baseline calibration.

inventors peaks no more than 3 percent higher than the baseline, whereas the average match quality conditional on a match only rises by a fraction of a percent. The positive incentive effects win out for any reasonable level of R&D subsidies, but the effects are small. Hence, R&D subsidies do not recover the losses from search frictions in the market for inventors and leave growth on the table.

4.3 Partial and General Equilibrium Effects of R&D subsidies

Finally, I use the model to shed light on the differences between the general and partial equilibrium effects of R&D subsidies. As in all Schumpeterian growth models, an increase in aggregate innovation increases the rate of creative destruction Δ,

Figure 8: The Impact of R&D Subsidies s_r on Match Quality and the Share of Matched Inventors

Notes: This figure plots the share of inventors who are matched and their average match quality conditional on being matched as R&D subsidies increase. To plot both variables on the same scale and to make interpretation easier, I normalize both quantities so that they are equal to 1.0 at the baseline calibration. Hence, a value of 1.01 corresponds to a 1 percent increase. The horizontal axis is the R&D subsidy as a share of GDP, so a value of 0.3 means that 30 percent of GDP is allocated to R&D subsidies.

which lowers the returns to innovation. Because this force is only present in general equilibrium, it will not be detected in case studies or quasi-experiments affecting a small share of firms. In my model, creative destruction also destroys match capital (the "Blackberry effect"), meaning that it not only worsens the returns to innovation but actually destroys some of the economy's innovative capacity. This side-effect of creative destruction is new to the literature.

I conduct two experiments to show how the general and partial equilibrium effects of R&D subsidies differ. First, I remove R&D subsidies altogether by setting $\tau_r = 0$ and re-solving for the new balanced growth path. This captures the long run general equilibrium effects of R&D subsidies as a whole. Second, I remove R&D subsidies for a small number of entrant firms and all their future innovations but leave the growth rate and distribution of product lines across z and q fixed.

[28] For the sake of simplicity, I assume that workers observe q but not whether the firm has access to subsidies, which keeps workers' policies the same: they move if and only if they meet a division that can offer them a strictly higher q.[29] I then re-solve for the values (V and N) and policies (h and r) of these product lines. I then find the long-run distribution of affected product lines across match qualities. As this is more involved, I leave that discussion to Appendix A.7.

In Table 4, I report the share of matched product lines, average match quality conditional on being matched, average product line-level search effort, average research good expenditures, average research good expenditures per *matched* product line, and average innovation rates ϕ for both experiments. I report all values relative to their average in the baseline economy. In general equilibrium, the impact of taking away R&D subsidies is small.[30] In partial equilibrium, however, taking away these subsidies lowers search effort and investment in the research good, lowering innovation for affected product lines by almost 20 percent in the long run. The effects of creative destruction operating through the classical incentive channel (captured in the rate of investment in r conditional on being matched) and the novel "Blackberry effect" (captured in the share of matched divisions) each account for about half of the decline of innovation.[31]

It is interesting to compare these results to Rao (2016), who finds that research effort responds to tax incentives with a lag, and that the long-run effects of R&D subsidies are larger than the short-run effects. My model can help rationalize these lags by introducing frictional search for inventors, but also shows that the long-run partial equilibrium effects of R&D subsidies may be much greater than their long-

[28] Both of these are long-term exercises and give firms enough time to adjust their search effort and settle into a new stationary distribution across match qualities q. Note the contrast with the calibration of γ in Section 3.1.1.

[29] Because I solve the model on a discrete grid, and because I implicitly assume a small moving cost to break ties in q, this has modest consequences for relatively small counterfactuals such as removing the R&D subsidy. It does mean that my results, as striking as they are, understate the long-run partial equilibrium impact of R&D subsidies.

[30] Growth declines from 2.0 to 1.96 percent. Innovation by entrants, which is not captured in Table 4, increases in response to the lower Δ and partly counteracts the decline in innovation by incumbents.

[31] This understates the degree to which introducing this new channel should shift our priors about the effects of R&D subsidies for two reasons. First part of the incentive effects come from forward-looking divisions anticipating that they will be less likely to match—hence, even the traditional channel through which creative destruction affects innovation will be understated in traditional growth models. Second, this comparison is at the product line level, but firms without an R&D subsidy will end up with many fewer product lines in partial equilibrium.

Table 4: General vs Partial Equilibrium Effects

Value	General Equilibrium	Partial Equilibrium
Matched share	$\sim$1.0	0.88
Average q	$\sim$1.0	$\sim$1.0
Average h	$\sim$1.0	0.76
Average r	0.94	0.76
Average r|Matched	0.95	0.86
Average ϕ	0.97	0.82

**Notes:* This table compares the long-run general and partial equilibrium effects of R&D subsidies. I normalize all values based on their equivalent in the baseline calibration, so a value of 0.95 indicates a 5 percent decrease relative to the baseline. "Matched share" is the share of product lines matched to an inventor. "Average q" is the average match quality conditional on being matched, "Average h" is the average search effort (number of headhunters hired) per product line, "Average r" is the average amount of the research good purchased per product line, "Average $r|$Matched" is the average amount of the research good purchased per *matched* product line, and "Average ϕ" is the average rate of innovation per product line. All values are unconditional on being matched unless specified. I report all values to two decimal places only. See Section 4.3 for details, and Appendix A.7 for a more detailed explanation of how I calculate the distribution of product lines across match qualities in the partial equilibrium exercise.

run general equilibrium effects.

5 Conclusion

In this paper I study a key step in the process of innovation: the allocation of inventors to firms. I find that inventor mobility across firms is associated with a large increase in the rate of innovation, and that such moves are quite frequent. Intuitively, the fact that mobility is associated with such large gains suggests that the quality of the match between an inventor and a firm is an important determinant of the rate of innovation, and the fact that inventors move so often suggests that the market for inventors is frictional and that inventors do not necessarily find the right match for their talents right away. I formalize these intuitions with a novel Schumpeterian growth model that features frictional search for inventors and a crucial role for the quality of the match between the inventor and the firm. I discipline this model with new moments from inventor-firm patent data, and I use the model to shed light on the role that the market for inventors plays in economy-wide growth. I then show that search frictions in the market for inventors are a significant drain on growth, and that conventional innovation policies do not recover this lost growth. This paper opens up the black box of research spending and takes the market for inventors seriously, and shows that doing so helps open up new avenues for research into growth and innovation policy.

References

Acemoglu, D., U. Akcigit, H. Alp, N. Bloom, and W. Kerr (2018). Innovation, reallocation, and growth. *American Economic Review 108(11)*, 3450–91.

Akcigit, U., S. Caicedo, E. Miguelez, S. Stantcheva, and V. Sterzi (2018). Dancing with the stars: Innovation through interactions. Technical report, National Bureau of Economic Research.

Akcigit, U. and N. Goldschlag (2023). Where Have All the "Creative Talents" Gone? Employment Dynamics of US Inventors.

Arkolakis, C., M. Peters, and S. Lee (2020). European immigrants and the united states' rise to the technological frontier.

Arundel, A. and I. Kabla (1998). What percentage of innovations are patented? empirical estimates for european firms. *Research Policy 27*(2), 127–141.

Atkeson, A. and A. Burstein (2019). Aggregate implications of innovation policy. *Journal of Political Economy 127*(6), 2625–2683.

Bell, A., R. Chetty, X. Jaravel, N. Petkova, and J. Van Reenen (2019). Who becomes an inventor in america? the importance of exposure to innovation. *The Quarterly Journal of Economics 134*(2), 647–713.

Blundell, R., R. Griffith, and F. Windmeijer (2002). Individual effects and dynamics in count data models. *Journal of econometrics 108*(1), 113–131.

Celik, M. A. (2023). Does the cream always rise to the top? the misallocation of talent in innovation. *Journal of Monetary Economics 133*, 105–128.

Corrado, C., C. Hulten, and D. Sichel (2009). Intangible capital and us economic growth. *Review of income and wealth 55*(3), 661–685.

Engbom, N. (2020). Misallocative growth. Manuscript.

Fujita, S. and G. Moscarini (2017). Recall and unemployment. *American Economic Review 102(7)*, 3875–3916.

Garcia-Macia, D., C.-T. Hsieh, and P. J. Klenow (2019). How destructive is innovation? *Econometrica 87*(5), 1507–1541.

Gavazza, A., S. Mongey, and G. L. Violante (2018). Aggregate recruiting intensity. *American Economic Review 108(8)*, 2088–2127.

Hong, S. and D. Werner (2020). Trends in Startups? Share of Jobs in the U.S. and Eighth District. *The Regional Economist 28*(1).

Hosios, A. J. (1990, 04). On The Efficiency of Matching and Related Models of Search and Unemployment. *The Review of Economic Studies 57*(2), 279–298.

Jones, B. F. (2009). The burden of knowledge and the ?death of the renaissance man?: Is innovation getting harder? *The Review of Economic Studies 76*(1), 283–317.

Kaas, L. and P. Kircher (2015). Efficient firm dynamics in a frictional labor market. *American Economic Review 105*(10), 3030–60.

Klette, T. J. and S. Kortum (2004). Innovating firms and aggregate innovation. *Econometrica 112(5)*, 986–1018.

Kogan, L., D. Papanikolaou, A. Seru, and N. Stoffman (2017, 03). Technological innovation, resource allocation, and growth. *The Quarterly Journal of Economics 132*(2), 665–712.

Koh, D., R. Santaeulàlia-Llopis, and Y. Zheng (2020). Labor share decline and intellectual property products capital. *Econometrica 88*(6), 2609–2628.

Lashkari, D. (2023). muratjmp. *Manuscript*.

Lentz, R. and D. T. Mortensen (2012). Labor market friction, firm heterogeneity, and aggregate employment and productivity. *Unpublished Manuscript*.

Peters, M. (2020). Heterogeneous markups, growth, and endogenous misallocation. *Econometrica 88(5)*, 2037–2073.

Peters, M. and C. Walsh (2022). Population growth and firm-product dynamics. Manuscript.

Peters, M. and F. Zilibotti (2021). Creative destruction, distance to frontier, and economic development. *The Economics of Creative Destruction - A Festschrift in Honor of Philippe Aghion and Peter Howitt*.

Prato, M. (2022). The global race for talent: Brain drain, knowledge transfer, and economic growth.

Rao, N. (2016). Do tax credits stimulate r&d spending? the effect of the r&d tax credit in its first decade. *Journal of Public Economics 140*(C), 1–12.

Shimer, R. (2006). On-the-job search and strategic bargaining. *European Economic Review 50*(4), 811–830.

Tyson, L. and G. Linden (2012). The corporate r&d tax credit and us innovation and competitiveness: Gauging the economic and fiscal effectiveness of the credit. *Center for American Progress*, 21–2.

US Bureau of Economic Analysis (2022). Gross domestic product. retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/GDP.

US Bureau of Labor Statistics (2022). Number of jobs, labor market experience, marital status, and health: Results from a national longitudinal study. retrieved from BLS; https://www.bls.gov/news.release/pdf/nlsoy.pdf.

van der Wouden, F. and D. L. Rigby (2020). Inventor mobility and productivity: a long-run perspective. *Industry and Innovation 28(6)*, 677–703.

Online Appendix

A Appendix: Solving the Model

I use a discretized version of the model, hence why I write sums rather than integrals in this section.

A.1 Pseudocode

- Step 0: Initialize the model with a guess for distributions F, F^V, F^N; general equilibrium objects π, g, Δ, w; values V, N; and policies r, h, e as described below.

  - Re-scale the $\hat{z}$ grid so $Z = 1$.
  - Guess that $\min[\nu M \cdot 0.5, 0.5]$ inventors are matched, i.e. that either half of inventors or half of all divisions are matched (this ensures that, no matter what I put for M and ν, I will not guess that more than a measure 1 of inventors or divisions are matched). Guess that matched divisions are uniformly distributed across the $\hat{z}, q$ grid, and that unmatched divisions are uniformly distributed across the $\hat{z}$ grid.
  - A reasonable first guess for V and N, conditional on the guess for the number of workers in production L^p, would be the present discounted value profits accruing to a division with productivity $\hat{z}$ assuming no innovation, drift from growth, or creative destruction. I increase these values by 10% for V to take into account that matched division have the option to innovate, and I increase them by a further multiplicative factor equal to $0.1 \cdot q/q_l$ for each value of q to take into account the greater value of a division with a good match.
  - Guess that 10 percent of non-inventor workers will work in producing the research good (this is r).
  - Guess that 5 percent of non-inventor workers will work in entry (this is e).

  - Guess that 0.1 percent of non-inventor workers will work in search (this is h), and that search effort is uniform for all divisions.

- Step 1: Holding fixed the distributions, update policies, values, and GE objects.

  - Step 1a: Holding fixed policies r, h, e and values V, N, solve for the GE terms Δ, w, π, g.
  - Step 1b: Re-scale every element of the q grid and ι by a common term κ until g is 2%[32], *unless* performing a counterfactual. Check if the growth rate had converged; i.e. check if it had already been 2% before rescaling.
  - Step 1c: Holding fixed values and GE terms, solve for policies r, h, e.
    * If divisions demand $R > 0.6M(1-\nu)$ units of labor as an input into research, scale down their demands by a factor $0.6M(1-\nu)/R$ such that no more than 60% of the non-inventor labor force is used for research.
  - Step 1d: Holding fixed policies and GE terms, update values V and N using an implicit step, and check if they have converged.
  - Step 1e: Repeat 1a-1d until values and the growth rate converge, or until the maximum number of inner-loop iterations is reached.

- Step 2: Update the distributions using the policies recovered in Step 1.

  - If $\sum_{\hat{z},q} F^V(\hat{z}, q) > M\nu$, i.e. there are more matched divisions than inventors, scale down the mass of matched divisions until only 90% of inventors are matched, and scale up the mass of unmatched divisions until the overall mass is again 1.0.

- Step 3: Repeat 1 and 2 until values, distributions, and the growth rate converge, or until the maximum number of outer-loop iterations is reached.

[32] I follow Engbom (2020) in holding fixed the growth rate and scaling the parameters to ensure that this growth rate is attained. I find, as he did, that the solution algorithm is more stable if I let the parameters vary but keep the growth rate fixed than vice versa.

A.2 Solving for GE objects

To calculate the rate of creative destruction Δ, I sum up innovative activity among matched divisions and entrants.

$$\Delta = e\iota/E^{\psi} + \int_{\hat{z},q} \phi(q) dF^{V}(\hat{z},q)$$

This allows me to get the growth rate g, which is $\Delta\lambda/(\sigma-1)$.

Note that both Δ and g can be scaled if we multiply ι and every value of q by a multiplicative $\kappa = 0.02/g$. Doing so would rescale innovation such that the growth rate would be 2 percent if policies were held fixed.

Wages and flow profits are as usual in CES models of monopolistic competition:

$$w = Y/(\mu l)$$

$$\pi = (\mu - 1)\hat{z}wL$$

A.3 Updating Policies

Solving for entry e is straightforward conditional on knowing the value of creative destruction $CD = \sum N(\lambda\hat{z})F(\hat{z})$. Recall the three key equations for this problem:

$$\phi_s = \iota/E^{\psi}$$

$$\phi_s CD = w$$

$$\phi_s e = E$$

Combining the first and second equation immediately yield:

$$E = \left(\frac{\iota CD}{w}\right)^{\frac{1}{\psi}}$$

Combining this with the first and third yields:

$$e = \frac{1}{\iota}\left(\frac{\iota CD}{w}\right)^{\frac{1-\psi}{\psi}}$$

Now I turn to the problem of the matched division choosing r. Conditional on CD, w, and τ_r, the division's effort level r can be solved in closed-form:

$$\max_r (1-(1-\tau_r)\chi)qr^{\gamma}CD-(1-\tau_r)wr$$

$$r=\left(\frac{(\gamma(1-(1-\tau_r)\chi)qCD)}{(1-\tau_r)w}\right)^{\frac{1}{1-\gamma}}$$

Finally, I calculate the number of workers hired as headhunters. Conditional on the value of meeting Ω and on the contact rate per "unit of matching" C_s, search effort can be solved as follows for unmatched and matched firms, respectively:

$$\eta h^{\eta-1}\Omega C_s = w(1-s_h)$$
$$h=\left(\frac{w(1-s_h)}{\eta\Omega C_s}\right)^{\frac{1}{\eta-1}}$$

$$\eta h(q)^{\eta-1}\Omega(q)C_s = w(1-s_h)$$

$$h(q)=\left(\frac{w(1-s_h)}{\eta\Omega(q)C_s}\right)^{\frac{1}{\eta-1}}$$

With r, h, e solved, labor used in production l is $M(1-\nu)$ minus labor used for all three other functions.

A.4 Updating Values

I use a discretized version of the HJB equations, with $\hat{z}$ and q on grids. I approximate the derivative of V and N with respect to $\hat{z}$ using a standard upwinding approach:

$$\frac{\partial V(\hat{z}_n)}{\partial \hat{z}} \sim \frac{V(\hat{z}_n)-V(\hat{z}_{n-1})}{\hat{z}_n-\hat{z}_{n-1}}$$

I stack V and N into a single vector S and construct an intensity matrix across states A that summarizes all the flows across these states. This matrix takes policies as given and incorporates the costs of taxation and wages incurred when a division or inventor moves across states. I denote S as the stacked vector of values and f as the flow utility coming from profits, research costs, and effort.

$$S_{t+\epsilon} = S_t + \epsilon(f + AS_t - (\rho - g)S_{t+\epsilon})$$

This can be rewritten as:

$$\underbrace{\left[\left(\frac{1}{\epsilon} + \rho - g\right) I - A\right]}_{T} S_{t+\epsilon} = \frac{1}{\epsilon} S_t + f$$

Finally we get the updated vector of values:

$$S_{t+\epsilon} = T^{-1}\left(\frac{1}{\epsilon} S_t + f\right)$$

I set a large value for ϵ, check for convergence, and *then* update the value of S by taking a convex combination of $S_{t+\epsilon}$ and S_t.

A.5 Updating Discretized Kolmogorov Forward Equations

It is sufficient to characterize the distribution of divisions across states $(\hat{z}, q)$—knowing this distribution automatically pins down the distribution of workers. Denoting G as the stacked vector of distributions across states and K as the transition matrix across states, we get:

$$G_{t+\epsilon} = G_t - \epsilon K' G_t$$

It is possible to solve for the stationary distribution directly by iterating on this equation repeatedly or by looking for the solution to:

$$K'G = 0$$

I find the following approach works well: first, scale every element of K' by some factor s such that the sum of absolute values in each row is much less than 1.0 (recall that only the diagonal elements of K' are negative). Next, add I to K'. Then look for the solution to:

$$(I + sK')G = G$$

$(I + sK')$ is a stochastic matrix where all elements are positive and the rows sum to 1, and I can find G using off-the-shelf routines for finding the stationary distribution of a stochastic process.

The matrix K used to update the distribution is different from the one used to update the values (A). First, a creatively destroyed division does not derive value from the new division created to replace it, but the "destroying" division does. I do not keep track of the "destroying" distribution in A, but I do in K. Second, K does not incorporate taxes.

Notice one slightly unusual thing about K: the elements of the distribution actually show up directly in the matching probabilities. To solve for G, I hold K fixed and update till G converges, then I recalculate K and re-solve for G. I effectively iterate on solving for K and G until both converge.

A.6 Extreme Values of $\hat{z}$

I solve the model on a discretized grid of $\hat{z}$, which does not have a natural upper or lower bound. In this section, I describe a few adjustments that I make to the model to deal with this discretization.

First, at the lowest point of the $\hat{z}$ grid, I set the terms $\partial V(\hat{z}, q)/\partial \hat{z}$ and $\partial N(\hat{z})/\partial \hat{z}$ to 0.

Second, I modify the value of creative destruction CD. If a division or unmatched inventor creatively destroys a rival division with $\hat{z} > \hat{z}_{\text{max}}$, the highest value of $\hat{z}$ on the grid, they spin off a new unmatched division with productivity $\epsilon \to 0$ greater than the rival, not λ greater. Hence, the value of CD becomes:

$$CD = \sum N(\min[\hat{z}_{\text{max}}, \lambda + \hat{z}])F(\hat{z})$$

Finally, when updating g, I setΔ to 0 for the highest value of the grid.

A.7 Partial Equilibrium Exercise: Solving for the Distribution

I define m_q as the share of affected divisions in the counterfactual at match quality q, where $q = 0$ denotes an unmatched division. I define x_q as the flow rate at which an inventor will be poached away from a division with match quality q. I define $p_{a,b}$ as the flow rate at which a division at match quality a will meet an inventor, draw match quality b, and successfully match. The term e_a is the rate of entry of new affected product lines as a share of the total.

The fraction of affected divisions that are unmatched must be such that inflows balance outflows. Inflows come from entry, matches breaking, inventors being poached, and firms expanding into new varieties and creating new unmatched divisions. Outflows come from creative destruction or divisions matching.

$$m_0 = m_0 + \underbrace{e_a + \sum_{q=1}^{10} m_q(\delta_m + \delta_i + x_q + \phi_q)}_{\text{Inflows}} - \underbrace{(\Delta + \sum_{q=1}^{10} p_{0,q})m_0}_{\text{Outflows}}$$

The fraction of affected divisions at match quality q must also be such that inflows balance outflows. Inflows come from other divisions at lower points on the quality distribution matching, and outflows come from creative destruction, poaching, and matches breaking.

$$m_q = m_q + \underbrace{\sum_{a=0}^{q-1} p_{a,q} m_a}_{\text{Inflows}} - \underbrace{(\Delta + \delta_m + \delta_i + x_q) m_q}_{\text{Outflows}}$$

I add one more constraint so that the terms m will sum to 1 and so that the optimizer can find e:

$$m_0 + \sum_{q=1}^{10} m_q = 1$$

I use a standard optimization routine to solve for all values of m and for e_a.

B Appendix: Deriving Growth and Drift Rates

In this section I offer heuristic explanations of how to calculate g and where the term $g(\sigma-1)\partial N(\hat{z})/\partial\hat{z}$ in the HJBs comes from. To do so, the following derivations about $\hat{z}$ will be useful.

$$\hat{z} = \left(\frac{z}{Z}\right)^{\sigma-1}$$

$$Z = \left(\int_{\hat{z}} z^{\sigma-1}F(z)\right)^{\frac{1}{\sigma-1}}$$

$$Z = \left(\int_{\hat{z}} \hat{z}Z^{\sigma-1}F(z)\right)^{\frac{1}{\sigma-1}}$$

$$1 = \left(\int_{\hat{z}} \hat{z}F(z)\right)^{\frac{1}{\sigma-1}} = \int_{\hat{z}} \hat{z}F(z)$$

The growth rate g can be calculated by calculating the percent change in Z over an interval of time divided by the length of the interval, and then taking the limit of that expression as the length of the interval goes to zero.

$$g = \lim_{\epsilon\to 0}\frac{1}{\epsilon}\frac{Z_{t+\epsilon}-Z_t}{Z_t}$$

Recall:

$$Z_t = \left(\int_z z^{\sigma-1}F_t(z)\right)^{\frac{1}{\sigma-1}} = Z\left(\int_{z_t}\frac{z^{\sigma-1}}{Z^{\sigma-1}}F_t(z)\right)^{\frac{1}{\sigma-1}} = \left(\int_z \hat{z}F_t(z)\right)^{\frac{1}{\sigma-1}}$$

At each point of the productivity grid, some share of the mass at that grid point jumps from $\hat{z}$ to $\lambda+\hat{z}$ at a rate of Δ. Over a time step ϵ, this means that:

$$Z_{t+\epsilon} = \left(\int_z (\hat{z}+\epsilon\Delta\lambda)F_t(z)\right)^{\frac{1}{\sigma-1}}$$

Dividing the numerator and denominator by Z_t, dropping time subscripts, and we get:

$$g = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \frac{\left(\int_z (\hat{z} + \epsilon\Delta\lambda) F_t(z)\right)^{\frac{1}{\sigma-1}} - 1}{1}$$

Simplifying a bit:

$$g = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \frac{\left(\int_z \hat{z} F_t(z) + \int_z \epsilon\Delta\lambda F_t(z)\right)^{\frac{1}{\sigma-1}} - 1}{1} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \frac{(1 + \epsilon\Delta\lambda)^{\frac{1}{\sigma-1}} - 1}{1}$$

Taking the limit as $\epsilon \to 0$ yields the expression in the paper:

$$g = \frac{\Delta\lambda}{\sigma - 1}$$

To understand where the term $g(\sigma - 1)\partial N(\hat{z})/\partial\hat{z}$ in the value functions comes from, I work with a simplified discrete-time version of the model without innovation. This can be written as:

$$N(Z_t, \hat{z}_t) = \epsilon\pi(Z_t, \hat{z}_t) + e^{-r\epsilon} N(Z_{t+\epsilon}, \hat{z}_{t+\epsilon})$$

Because π scales multiplicatively in Z and $\hat{z}$, this can be rewritten with a constant term $\tilde{N}$:

$$\tilde{N} Z_t \hat{z}_t = \epsilon\pi Z_t \hat{z}_t + e^{-r\epsilon} \tilde{N} Z_{t+\epsilon} \hat{z}_{t+\epsilon}$$

Dividing both sides by Z_t and using the fact that $Z_{t+\epsilon} = Z_t e^{\epsilon g}$:

$$\tilde{N} \hat{z}_t = \epsilon\pi \hat{z}_t + e^{(g-r)\epsilon} \tilde{N} \hat{z}_{t+\epsilon}$$

Note that a standard Euler condition gives us that $r = \rho + g$, simplifying terms a bit. Subtracting $\tilde{N}\hat{z}_t e^{-\rho\epsilon}$ from both sides and dividing by ϵ we get:

$$\frac{\tilde{N}\hat{z}_t \left(1 - e^{-\rho\epsilon}\right)}{\epsilon} = \pi\hat{z}_t + \frac{e^{-\rho\epsilon} \tilde{N} (\hat{z}_{t+\epsilon} - \hat{z}_t)}{\epsilon}$$

Taking the limit of both sides as $\epsilon \to 0$ we get:

$$\rho\tilde{N}\hat{z}_t = \pi\hat{z}_t + \lim_{\epsilon \to 0} \frac{e^{-\rho\epsilon} \tilde{N} (\hat{z}_{t+\epsilon} - \hat{z}_t)}{\epsilon}$$

To simplify the right hand side, recall $\hat{z}_t = (z_t/Z_t)^{\sigma-1}$, meaning $\hat{z}_{t+\epsilon} =$

$(z_t/(Z_te^{g\epsilon}))^{\sigma-1} = \hat{z}_t e^{(1-\sigma)g\epsilon}$. This lets us rearrange and get:

$$\rho\tilde{N} = \pi + \lim_{\epsilon\to 0} \frac{e^{-\rho\epsilon}\tilde{N}(e^{(1-\sigma)g\epsilon} - 1)}{\epsilon}$$

This simplifies to:

$$\rho\tilde{N} = \pi - g(\sigma-1)N$$

And recall that $N = \tilde{N}\hat{z}$, so $N' = \tilde{N}$.

C Appendix: Properties of the Equilibrium

In this section, I provide some more plots to help illustrate the properties of the equilibrium.

In Figure 9, I illustrate the distribution of product lines across relative productivities $\hat{z}$, and also break them out into matched and unmatched. There are two main takeaways from this first plot: First, the vast majority of product lines are unmatched because there are far fewer inventors than product lines. Second, the distribution of match qualities among unmatched product lines is very slightly more right-heavy than for matched lines because creative destruction both raises the relative productivity of a product line and breaks any existing match at that line. This is also a consequence of the fact that I only model creative destruction, not own-innovation.

In Figure 10, I display the joint distribution of *matched* product lines across both relative productivity $\hat{z}$ and match quality q. Given what we see in Figure 4 and Figure 9, it is not surprising that matched productivity lines tend to have high match qualities and low relative productivities.

In Figure 11, I plot the intensity of search effort (as measured by the mass of headhunters) across matched divisions with different match qualities q, normalized so that search effort for an unmatched division is 1.0. All matched divisions devote far less effort to search than unmatched ones, and search effort is monotone decreasing in match quality q. Those matched divisions with the highest level of match quality $q = q_h$ do not search at all as they cannot replace their current inventor with a better one.

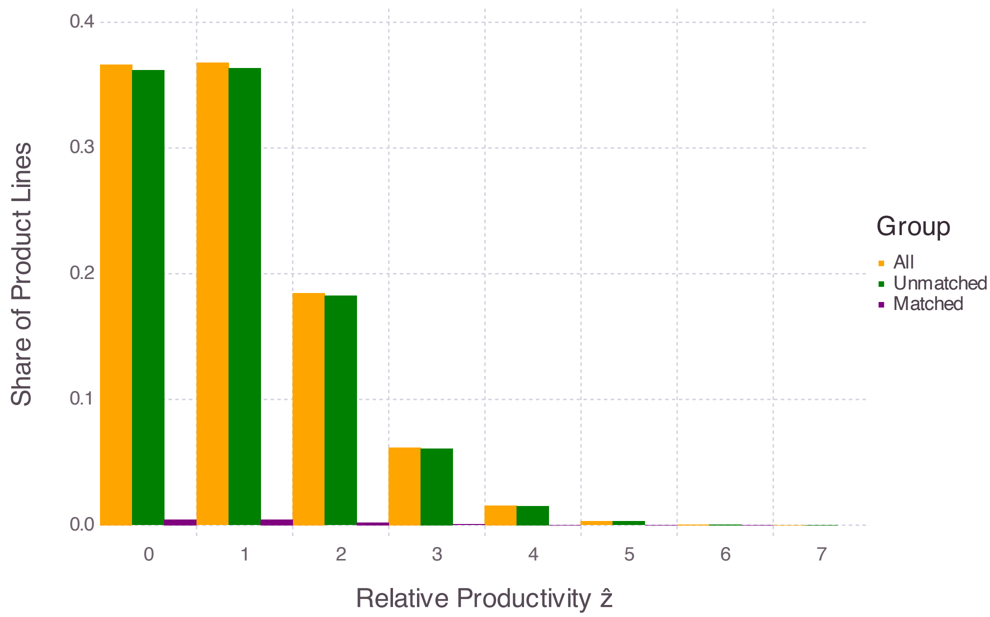


Figure 9: Distribution of Product Lines Across Relative Productivities

Notes: This figure plots the distribution of product lines in relative productivity ($\hat{z}$) space. I solve the model on a discrete grid of relative productivities. The horizontal axis is the relative productivity, and the vertical axis is the share of product lines with that productivity. In each $\hat{z}$ bin, the share of matched and unmatched lines add up to the share of all lines by construction. The $\hat{z}$ grid actually goes all the way to 19, but there is so little mass in those extreme gridpoints that I do not plot them.

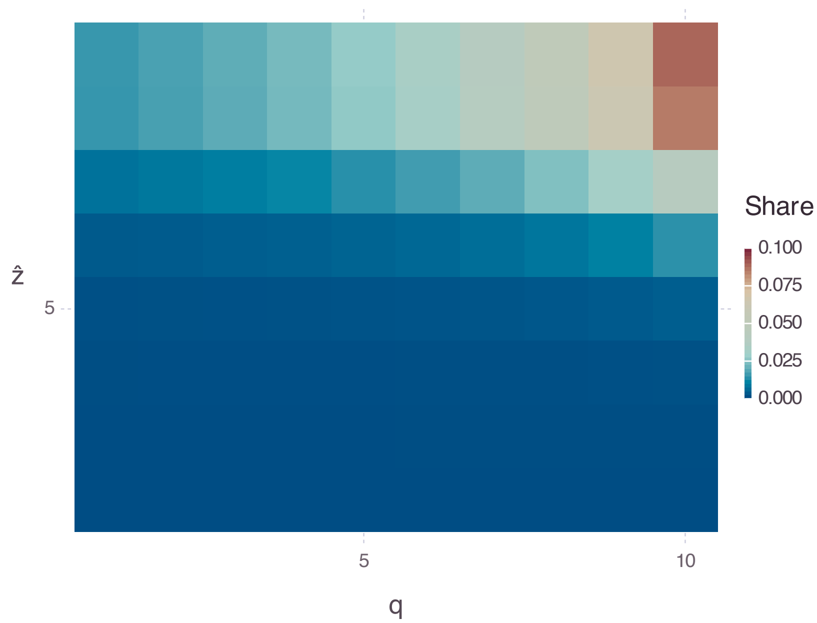


Figure 10: Joint Distribution of Matched Product Lines Across $\hat{z}$ and q

Notes: This figure plots the joint distribution of matched product lines in relative productivity ($\hat{z}$) and match quality (q) space. The horizontal axis is relative productivity, the vertical axis is match quality, and the color indicates the share of matched product lines in that cell. As in Figure 9, I cut off the $\hat{z}$ axis as there is vanishingly little mass in the highest relative productivities.

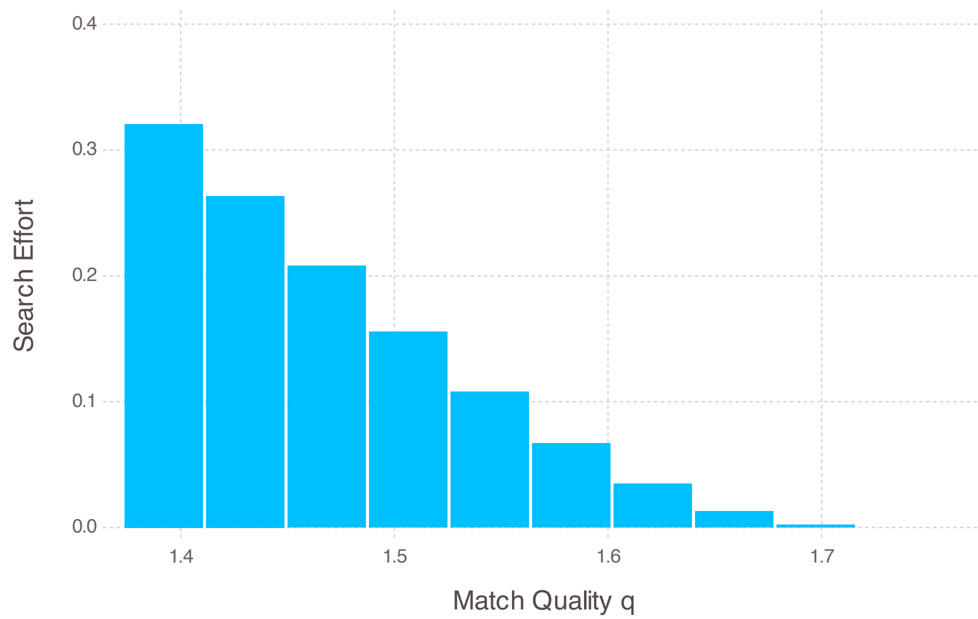


Figure 11: Search Effort for Matched Divisions

Notes: This figure plots the quantity of "headhunter" labor h hired by matched divisions with different relative productivities q. I normalize the graph so that the quantity of "headhunters" hired by an unmatched division is 1.0. The divisions with the highest match quality exert no search effort.